POKHARA UNIVERSITY

Level: Bachelor | Semester: Fall | Year: 2021

Programme: BBA/BI/TT | Full Marks: 100

Course: Business Mathematics II | Pass Marks: 45

Time: 3 hrs.

Candidates are required to answer in their own words as far as practicable. The figures in the margin indicate full marks.

Section "A"

Very Short Answer Questions

Attempt all the questions. [10×2]

1. Evaluate: $\lim_{x\to4}\frac{x^3-64}{x^2-16}$.
2. Find $\frac{dy}{dx}$ if $y=\sqrt{u}$ and $u = x^4$.
3. Write a formula for elasticity of demand and elasticity of supply. Also write a condition for elastic demand.
4. Check whether $f(x) = -x^2-5x+20$ is increasing or decreasing at $x= -10$.
5. A firm has the demand function $P=14-2q$ and the cost function $C=q^2+2q$. Find MC and MR.
6. Draw a graph of inequality $x+2y\geq 5$.
7. Solve $ydx - xdy = xydx$
8. The marginal revenue for selling x items is given by $MR = 45-14x-x^3$. Find the total revenue function.
9. Verify Euler's theorem for $u=x^2+xy+y^2$.
10. Evaluate $\int xe^{-4x}dx$.

Section "B"

Descriptive Answer Questions

Attempt **any six** questions. [6×10]

11. a) If a demand function is $p = \frac{33}{x-8}$ where p is the price and x, the quantity, show by using the concept of limit that the demand increases indefinitely as the price falls. Also show that the total revenue approaches a definite value when the quantity demanded increases indefinitely.

    b) A function f(x) is defined as;

$$f(x) = \begin{cases} 2-x^2 & for\ x<2 \\ 3 & for\ x=2 \\ x-4 & for\ x>2 \end{cases}$$

Does lim f(x) exist? Is the function contineous at x=2? Why? Justify.

12. a) The total cost and total revenue functions for a product are C(q)=500 +100q+0.5q^2 and R(q)=500q.

i. Using marginal approach determine the profit maximizing level of output.

ii. What is maximum profit.

b) Find the maximum and minimum values of the function u=f(x, y)= x^2 - 3xy-5y^2 when 2x+3y=6.

13. a) A product with a large advertising budget has its sells given by S=$\frac{500}{t+2}$ - $\frac{1000}{(t+2)^2}$, where t is the number of months the product has been on the marker.

i. Find the rate of change of sells at any time t.

ii. What is the rate of change of sales at t=2?

b) If U= log(x^2+y^2+z^2) then prove that x.u_x+y.u_y+z.v_z=2

14. a) If AR. and MR. denote the average and marginal revenue at any output level, show that the elasticity of demand is equal to $\frac{AR.}{AR.-MR.}$. Verify this relation for $p = a + bx$, where p is price and x is the quantity.

b) The demand function of a certain commodity is P=$\frac{1}{3}$ Q^2 -10Q +75. Find the value of Q and the corresponding value of P that maximize the revenue.

15. a) Given the production function P=K$L^{\alpha}C^{\beta}$ where P is product, L is labour, C is capital and K, $\propto$ and β are constants, find dp.

b) Find the first and second order total derivative of u with respect to t where u=4x^2+4xy; x=2t^3+2 y=1-2t.

16. a) Find the general and particular solution of the differential equation $\frac{d^2y}{dx^2}$= 7x-5, $f'(3)$=5, f(- 2)=10.

b) Integrate (i)$\int \frac{1}{x(x-1)} \log\left(1-\frac{1}{x}\right)dx$. (ii) $\int x^2 e^{-2x}$ dx.

17. a) Evaluate $\int_0^{\infty} \frac{8x}{(2x^2+5)^{3/2}}$ dx.

b) A product's demand function is P=16-x^2 and its supply function is P_s=2x^2+4 where P is the price per unit in dollars and x is the number of units. Find the consumer surplus and producer surplus.

Section "C"

Case Analysis

18. a) Read the case situation given below and answer the questions that follow: [10]

Par Inc is a small manufacturer of golf equipment and supplies whose management has decided to move into the market for medium and high priced golf bags. Par's distributor is enthusiastic about new product line and has agreed to buy all the golf bags Par produces over the next three months.

After a thorough investigation of the steps involved in manufacturing a golf bag, management determines that each golf bag produced will require the following operations:

- Cutting and dyeing the material
- Sewing
- Finishing (inserting umbrella holder, club separators, etc)
- Inspection and packaging

The director of manufacturing analyzed each of the operations and concluded that if the company produces a medium priced standard model, each bag will require 7/10 hour in the cutting and dyeing department, ½ hour in the sewing department, 1 hour in the finishing department and 1/10 hour in the inspection and packaging department. The more expensive deluxe model will require one hour for cutting and dyeing, 5/6 hour for sewing, 2/3 hours for finishing and ¼ hour for inspection and packaging. Par's production is constrained by a limited number of hours available in each department. After studying departmental workload projections, the director of manufacturing estimates that 630 hours for cutting and dyeing, 600 hours for sewing, 708 hours for finishing and 135 hours for inspection and packaging will be available for the production of golf bags for the next three months.

The accounting department analyzed the data, assigned all relevant variable cost, and arrived at prices for the both bags that will result in a profit contribution of $10 for every standard bag and $9 for every deluxe bag produced.

i. Develop a mathematical model of the Par inc., problem that can be used to determine the number of standard bags and number of deluxe bags to produce in order to maximize total profit contribution.

ii. Find an optimum solution.

b) The manager of a plant has been instructed to hire and train additional employees to manufacturer a new product. She must hire a sufficient number of new employees so that within 30 days the will be producing 2500 units of the product each day.

Because a new employee must learn an assigned task, production will increase with training. Suppose that research on similar projects indicates that production increase according to the learning curve, so that for the average employee, the rate of production per day is given by $\frac{dN}{dt} = be^{-at}$ where N is number of units produced per day after t days of training and a and b are constants that depend on the project. Because of experience with a similar project, the manager expects the rate for this project to be $\frac{dN}{dt} = 2.5e^{-0.05t}$. The manager tested her training program with 5 employees and learned that the average employee could produce 11 units per day after 5 days of training. On the basis of this information, she must decide how many employees to hire and begin to train so that a month from now they will be producing 2500 units of the product per day. She estimates that it will take her 10 days to hire the employees, and thus she will have 15 days remaining to train them. She also expects a 10% of attrition during this period.

How many employees would you advise the plant manager to hire? Check your advice by answering the following questions: [10]

i. Use the expected rate of production and results of the manager's test to find the function relating N and t, i.e. $N = N(t)$.

ii. Find the number of units the average employee can produce after 15 days of training. How many such employees would be needed to maintain a production rate of 2500 units per day?

iii. Explain how you would revise this last result to account for the expected 10% attrition rate. How many new employees should the manager hire?