POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year: 2021
Programme: BBA/BI/TT | Full Marks: 100
Course: Business Mathematics II | Pass Marks: 45
Time: 3 hrs.

Candidates are required to answer in their own words as far as practicable. The figures in the margin indicate full marks.

Section "A"

Very Short Answer Questions

Attempt all the questions. [10×2]

1. Evaluate: $\lim_{x \to \infty} \frac{\sqrt{x^2-1}}{2x+1}$.
2. Check whether the function $f(x) = -x^3 + 4x^2 + 15$ is concave upward or downward at $x = 5$.
3. Find the derivative of $y = x^4 + 3x^3 + 7$ *with respect to* x^2 *at* $x = 3$.
4. Solve: $ydx - xdy = 0$.
5. Evaluate: $\int \frac{x+2}{x-2} dx$.
6. Suppose that the demand equation for a certain commodity is $q = 200 - 5p$. Find the elasticity of demand in terms of p.
7. If the total Cost function is $C = 30 + 10x + 5x^2$, find the average cost and marginal cost at $x = 10$.
8. Find $\frac{dy}{dx}$ if $x^2+y^2 = 5xy$.
9. If $f(x, y) = x^3 + y^3 - 3x - 27y + 24$, find the critical points.
10. Draw the graph of $2x + 3y \leq 6$.

Section "B"

Descriptive Answer Questions

Attempt **any six** questions. [6×10]

11. a) Evaluate: $\lim_{x \to 4} \frac{x^2-16}{\sqrt{3x+4}-4}$

    b) A function $y = f(x)$ is defined by

$$f(x) = \begin{cases} Ax^2 + 5x - 9 & for\ x < 1 \\ B & for\ x = 1 \\ (3-x)(A-2x) & for\ x > 1 \end{cases}$$

    If it is continuous for all x, find A and B.

12. a) The population of a country is estimated by the function $P = 125e^{0.035t}$ where P equals the population (in millions) and t equals time measured in years since 2020.

i) What is the population expected to equal in the year 2030?

ii) Determine the expression for the instantaneous rate of change in the population.

b) If AR and MR denotes the average and marginal revenue at any output level, show that the elasticity of demand is equal to $\frac{AR}{AR-MR}$.

13. a) Sales of post-paid mobile SIM cards the expected to vary with time, so that the cumulative total sold at 't' weeks after the sales is launched by NTC, s(t), given by the following equation $s(t) = \frac{3000}{1+500e^{-0.3t}}$.

Find and expression for the weekly rate of change in cumulative sales.

b) Find the maximum and minimum values of the function $f(x) = x + \frac{1}{x}$.

14. a) The production function is $P = KL^{\alpha} C^{\beta}$ where P is the product, L is labor, C is capital, K, α and β are constants. Find dP.

b) Find the first and second order total derivatives $\left[\frac{dU}{dt}, \frac{d^2U}{dt^2}\right]$ if

$$U = 2x^2 + xy + 4y^2, \quad x = 2t+1, \quad y = t+1.$$

15. a) If $u = \log\sqrt{x^2+y^2+z^2}$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{x^2+y^2+z^2}$

b) Using x skilled workers and y unskilled workers, a manufacturer can produce $Q(x, y) = 20x^2y^2$ units per day. Currently there are 20 skilled workers and 35 unskilled workers on the job. Use partial derivatives to answer the following:

i) By how much will the daily production level change if one more skilled worker is added to the current work force?

ii) By how much will the daily production level change if one more unskilled worker is added to the current work force?

16. Integrate **any two** of the following:

a) $\int_0^{ln2} xe^x dx$

b) $\int \frac{1}{x(5+logx)} dx$

c) $\int \frac{dx}{x+\sqrt{x}}$.

17. a) Find the general and particular solution for the differential equation:

$$\frac{d^2y}{dx^2} = 6x + 18, \quad f'(5) = -10, \quad f(2) = 30$$

b) The demand and supply function under perfect competition are $P_d = 16 - x^2$ and $P_s = 2x^2 + 4$ respectively. Find the market price, consumer's surplus and producer's surplus.

Section "C"

Case Analysis

18. a) Par, Inc. is a manufacturer of golf equipment and supplies whose management has decided to move into the market for medium-and high-priced golf bags. Par's distributor is enthusiastic about the new product line and has agreed to buy all the golf bags that Par produces over the next three months.

After a thorough investigation of the steps involved in manufacturing a golf bag, management determined that each golf bag produced will require the following operations:

i. Cutting and dyeing the material
ii. Sewing
iii. Finishing
iv. Inspection and packaging

The director of manufacturing analyzed each of the operations and concluded that if the company produces a medium-priced standard model, each bag will require $\frac{7}{10}$ hour in the cutting dyeing department, $\frac{1}{2}$ hour in the sewing department, 1 hour in the finishing department and $\frac{1}{10}$ hour in the inspection and packaging department. The more expensive deluxe model will require 1 hour for cutting and dyeing, $\frac{5}{6}$ hour for sewing, $\frac{2}{3}$ hour for finishing and $\frac{1}{4}$ hour for inspection and packaging.

Par's production is constrained by a limited number of hours available in each department.

After studying departmental workload projections, the director of manufacturing estimates that 630 hours for cutting and dyeing, 600 hours for sewing, 708 hours for finishing, and 135 hours for inspection and packaging will be available for production of golf bags during the next three months.

The Accounting department analyzed the production data, assigned all relevant variable costs, and arrived at prices for both bags that will result in a profit contribution of $10 for every standard bag and $9 for every deluxe bag produced.

[10]

i) Develop a mathematical model of the Par, Inc., problem that can be used to determine the number of standard bags and the number of deluxe bags to produce in order to maximize total profit contribution.

ii) Find an optimum solution.

b) Retails often sell different brands of competing products. Depending on the joint demand for the products, the retailer may be able to set prices that regulate demand and, therefore, influence profits. Suppose HOME-ALL, Inc., a national chain of home improvement retailers, sells two competing brands of interior flat paint, En-Dure 100 and Croyle & James, which the chain purchases for $8 per gallon and $10 per gallon, respectively. HOME-ALL's research department has determined the following two monthly demand equations for these paints:

$$D = 120 - 40d + 20c \quad \text{and} \quad C = 680 + 30d - 40c$$

where D is hundreds of gallons of En-Dure 100 demanded at $d per gallon and C is hundreds of gallons of Croyle & James demanded at $c per gallon. For what prices should HOME-ALL sell these paints in order to maximize its monthly profit on these items?

To answer this question, complete the following. [10]

i) Recall that revenue is a product's selling price per item times the number of items sold. With this in mind, formulate HOME-ALL's total revenue function for the two paints as a function of their prices

ii) Form HOME-ALL's profit function for the two paints (in terms of their selling prices).

iii) Determine the price of each type of paint that will maximize HOME-ALL's profit.

iv) Write a brief report to management that details your pricing recommendations and justifies them.